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LETTER TO THE EDITOR

Concentration effects in random ballistic deposition with restructuring

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Abstract. The influence of a finite concentration of falling particles in random ballistic deposition was studied using a simple two-dimensional model on a square lattice which includes some restructuring. The scaling properties of the surface of the deposit were studied as a function of the concentration. The exponent β, which describes how the surface thickness grows with the height of the deposit (for an extremely large deposit), varies from $\frac{1}{4}$ (for small concentrations) to $\frac{1}{3}$ (for large concentrations).

The deposition of particles on a surface is a phenomenon of both theoretical and practical interest. The simulation of such processes was pioneered by Vold [1] almost 30 years ago. More recently, new scientific interest has developed in random deposition, following the general interest in irreversible growth and aggregation phenomena [2] which was initiated by the introduction of a very simple model by Witten and Sander for diffusion-limited aggregation [3]. Random deposition can be viewed as a particle-cluster aggregation mechanism in which incoming particles stick one after another to a growing aggregate (the deposit) in a particular 'strip' geometry. Particles are released from above and stick to a horizontal basal plane. It appears that the structure of the deposit strongly depends on the nature of the trajectory followed by the incoming particles. While the deposit has the characteristic fractal structure of the Witten-Sander model in the case of Brownian trajectories [4], it is uniform (i.e. characterised by a constant density) in the case of vertical random trajectories (ballistic case) [5, 6] or in the case of the Eden model [7], where there is no trajectory and particles stick at random on any surface site of the deposit.

For the ballistic deposition and Eden models, it has been shown that the surface of the deposit exhibits a self-affine [8] fractal geometry which can be described in terms of the scaling form [5]

$$\xi = l^{\alpha} f(h/l^{\alpha/\beta}) \tag{1}$$

where ξ is the variance in the surface height, h is the mean height of the deposit and l is the lateral size of the strip (periodic boundary conditions are generally used at the edge of the strip). The scaling function $f(x)$ tends to a constant value when x tends to infinity and behaves as x^{β} when x tends to zero. This implies that ξ behaves as l^{α} for large h and as h^{β} for large l. In two dimensions numerical simulations give $\alpha = \frac{1}{2}$ and β about $\frac{1}{3}$ for both the ballistic [5, 6] and the Eden model [7]. In the case of ballistic deposition, it has been shown that the value of the β exponent becomes $\frac{1}{4}$

when surface diffusion [9] or complete restructuring [10] is included. In the latter case, the added particle is allowed to reach the nearest local minimum on the surface as soon as it first contacts the deposit. It appears the result ($\beta = \frac{1}{4}$) for ballistic deposition with restructuring can be very well accounted for by an analytical approach of Edwards and Wilkinson [11], while the value for β without restructuring ($\beta = \frac{1}{3}$) can be accounted for by an extension of this theory, due to Kardar *et al* [12], which includes the possibility of lateral growth.

In all the previous approaches, the limit of a very low particle concentration in the incoming flux was implicitly assumed since particles were added to the deposit one at a time. Very recently, it has been shown that, without restructuring, a finite concentration does not affect the scaling behaviour of the surface [13]. In this letter, we investigate the effect of a finite concentration together with particle restructuring in a two-dimensional model for random ballistic deposition. We show that, when the concentration is increased, the restructuring appears to be progressively 'frustrated' and a change in the scaling properties is obtained: the exponent β varies from $\frac{1}{4}$ (low concentration) to $\frac{1}{3}$ (high concentration).

Our simulation is performed on a square lattice. The deposit is grown within a strip of width of l lattice units, with periodic boundary conditions at the edge of the strip, starting from a horizontal basal line originally filled with l 'ground' particles. Before starting the simulation, 'rain' particles are deposited randomly, with concentration ρ, in the strip above the basal line. In this process, each site is visited and occupied with a rain particle with a probability equal to ρ. The height of the strip which must be investigated is equal to $h/(1-\rho)$ where h is the desired mean height for the final deposit. During the course of the simulation all 'rain' particles fall and are progressively transformed into 'ground' particles according to the following iterative method.

At each iteration all rain particles are investigated, row by row, starting from the lowest one. When a given particle is visited, an attempt is made to move it downwards by one lattice spacing. If there were no restriction coming from the presence of an immobile ground below, this motion would always be possible and it would result in a uniform rain, all particles falling vertically with the same velocity. This motion can be performed when there is a free site below (case A of figure 1). When the site below is occupied the two sites located on its right and on its left on the same row (i.e. the row immediately below the rain particle) are investigated. If these two sites are free, the rain particle is moved to one of these positions with equal probability (case B). If only one of these positions is free, the rain particle is moved to this free position

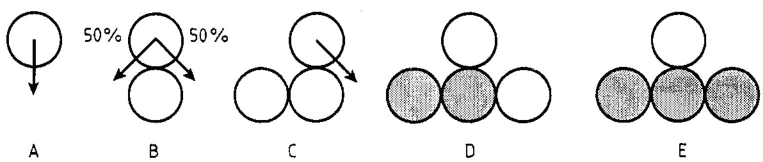

Figure 1. Sketch of the different possible motions for a rain particle. In case A, the site below is free: the particle moves vertically by one lattice spacing. In case B, the site below is occupied but the two adjacent sites on the same row are free: the particle slides along a diagonal either to the right or to the left (at random). In case C, the site below is occupied and only one adjacent site is free: the particle slides to this site. In case D, the three sites below are occupied by particles which are not all part of the ground: the particle stays on place and remains alive. In case E, the three sites below belong to the ground: the particle dies and becomes part of the ground.

(case C). If the three sites below are occupied, the rain particle stays on place and, if at least one of these sites is occupied by a rain particle, it awaits a further iteration to have a chance to move (case D). If the three sites below are all occupied by ground particles, the rain particle 'dies': i.e. it becomes a 'ground' particle and will not be visited in the future iterations (case E). When all rain particles have been investigated (up to the top of the rain) the next iteration starts, etc.

Figure 2 shows four typical examples ($\rho = 0.2$, 0.4, 0.6, 0.8) in which a deposit of size 128×128 has been built. Only the top rows (those not fully occupied) are shown. In this figure the ground particles are black while the rain particles are white or grey. The grey particles, or 'mud' particles, are rain particles whose vertical motion has been

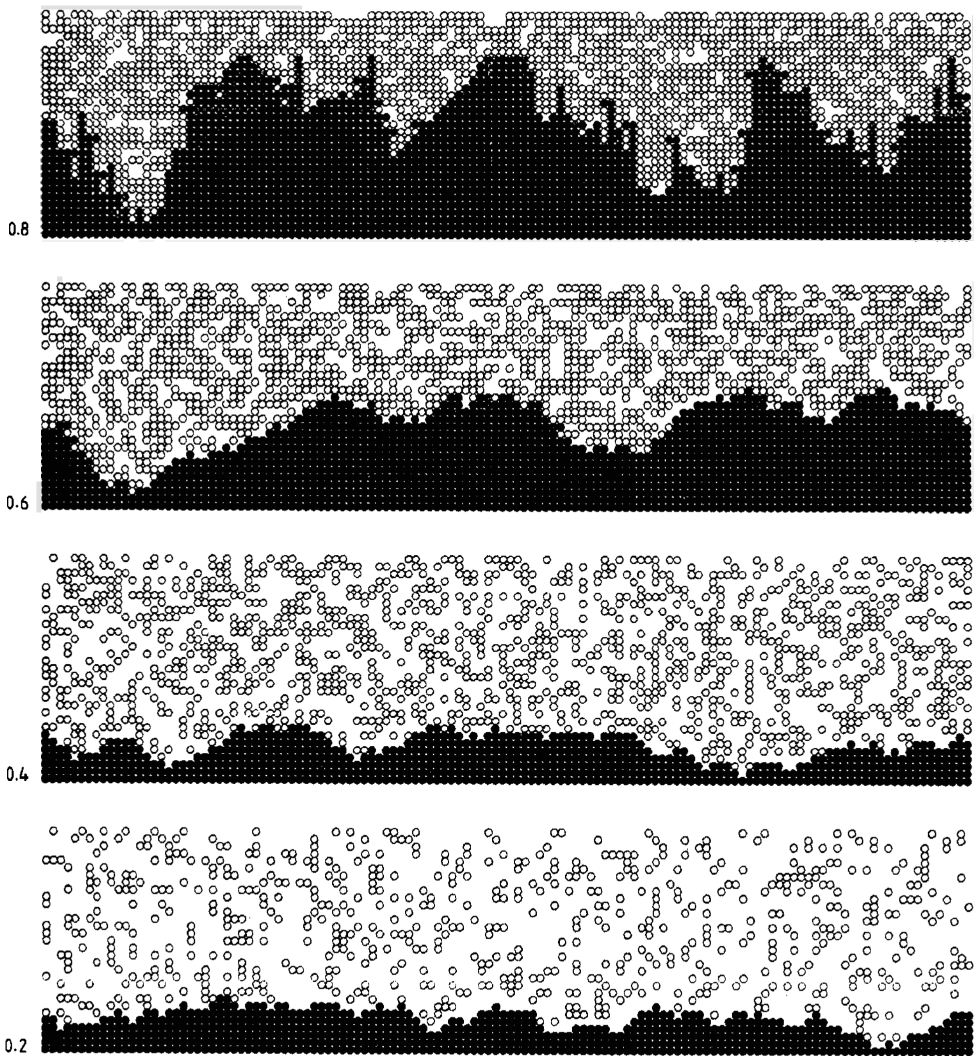

Figure 2. Typical deposits obtained for $\rho = 0.2$, 0.4, 0.6 and 0.8. These simulations were performed with $l = h = 256$. Only the top rows of the deposit (those containing surface sites) are shown. Ground particles are black and regular rain particles are white. The rain particles which were disturbed in their vertical motion are grey.

perturbed (having encountered cases B, C or D in the past). Mud particles represent rain sites that are correlated to the ground. The total number of mud particles (the 'mass' of the mud), divided by l, is the mud thickness, which is another characteristic length in our model. In the low concentration limit, the model becomes identical to a lattice rain model with restructuring. In this limit, as soon as a falling particle contacts the deposit, it slides along the deposit surface to reach a local minimum, and this restructuring is completely achieved before another particle is deposited, exactly as in the multi-restructuring models already introduced [10]. When the concentration increases, the restructuring motion slows down due to the presence of other particles arriving in the vicinity. In the limit of very high concentration, the interface consists in a series of pyramidal 'hills', whose 'valleys' are initiated by the presence of low density regions in the concentrated rain. The valleys, which are randomly distributed on the surface of the deposit, are seeds for further growth and one can argue that the growing mechanism becomes quite similar to that of the Eden model [7]. In this limit, the amount of mud particles becomes quite large and these particles slide along the sides of the hills before becoming part of the ground.

We were able to reach $l = 2048$ and we have checked the scaling form (1) with $\alpha = 0.5$ for all concentrations. The exponent β can be estimated from the curves of figure 3, where the logarithm of the surface thickness $\ln(\xi)$ has been plotted as a function of the logarithm of the height of the deposit $\ln(h)$, up to $h = 1024$, for different values of the concentration, $\rho = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, after averaging the results over 40 runs. For low concentrations ($\rho < 0.3$), a linear behaviour, followed by beginnings of a saturation process (as expected from the scaling form (1)), is seen. For $\rho = 0.1$, the estimated slope in the linear regime is 0.23, only slightly smaller than the value $\frac{1}{4}$ already recovered in previous simulations [10] and expected theoretically [11] for ρ tending to zero. The small discrepancy can be attributed to the influence of the saturation. As ρ is increased, a small h linear regime develops in which ξ varies with the square root of h. Such trivial scaling, which is also present for small concentrations and was observed in previous simulations, corresponds to the regime where h is of the order of ξ, so that the basal line has a direct influence on the shape

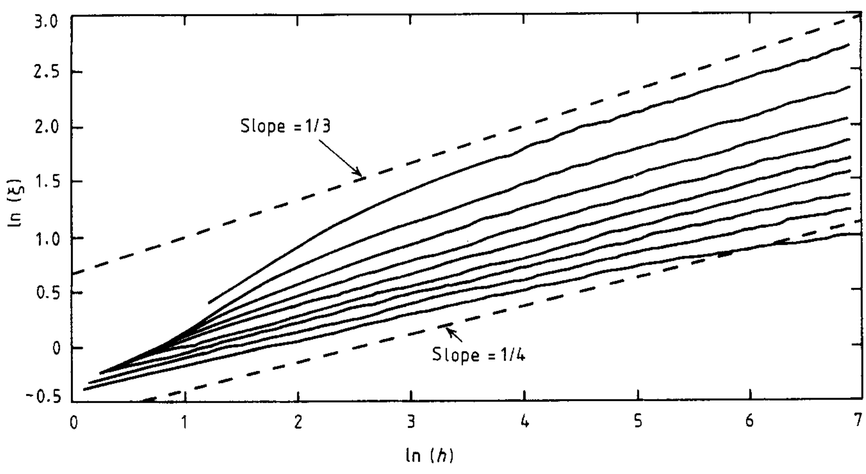

Figure 3. Surface thickness, ξ, of the surface as a function of the mean height of the deposit, h (log–log plot) for $\rho = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. ξ has been averaged over 40 simulations with $l = 2048$ and h varying up to 1024.

on the interface. This trivial scaling exists for heights up to about $1/(1-\rho)$, and this explains why it becomes more and more apparent as ρ increases. For larger h, there is a crossover to another linear regime from which the β exponent can be estimated. The saturation is pushed to larger heights when ρ is increased. The estimated values of β are reported in table 1 as a function of ρ. The results are consistent with a β exponent varying from $\beta=\frac{1}{4}$, for small ρ values, to $\beta=\frac{1}{3}$, for large ρ values.

In the same series of calculations, we have also evaluated the mass of the mud, m (we recall that this is the number of rain particles which were disturbed in their vertical motion: the grey particles of figure 2). The results are reported in figure 4, where we have plotted $\ln(m)$ as a function of $\ln(1-\rho)$ for different h values. These results suggest that, when h increases, m saturates to a value m_{sat}, which diverges when ρ tends to 1. From the figure we expect a power law divergence of the form:

$$m_{\text{sat}}\alpha(1-\rho)^{-\mu}$$

with μ approximately equal to 2. The saturation of m to a value independent of h is expected from the general result that the system reaches a steady state for sufficiently large h. However if must be noticed that the saturation of m arises well before the saturation of ξ. The divergence of m_{sat} when ρ approaches 1 corresponds to a divergence of the mud thickness m/l. In this limit, it can be argued that mud particles play the role of a screen for the growth process, so that the growth is no longer governed by outside (as in the regular ballistic model), but by the surface itself (as in the Eden model). This might explain why the exponent $\beta=\frac{1}{3}$ is recovered in this limit.

These results suggest that all the characteristics of the ballistic deposition vary smoothly with concentration. There is no particular concentration threshold, such as

Table 1. Values of the exponent β, estimated from the curves of figure 3. The absolute error on β is of order of 0.02.

ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β	0.23	0.24	0.25	0.265	0.275	0.285	0.30	0.31	0.325

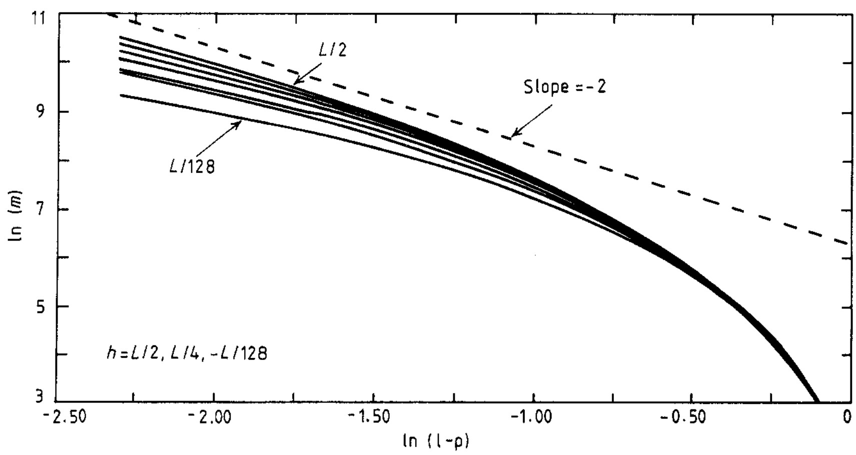

Figure 4. Number of rain particles which were disturbed in their vertical motion, m, as a function of $1-\rho$ (log-log plot) for constant h values, $h=L/128$, $L/64$, $L/32$, $L/16$, $L/8$, $L/4$, $L/2$.

the percolation threshold, at which one might observe a change in the structure of the deposit. This could be due to the fact that the model is built on a lattice. We are presently extending this study off lattice. Although these simulations have been presented in terms of a deposition process, a more relatistic physical realisation of the model, which might be amenable to experimental investigation, would involve the sweeping of particles which have been randomly deposited onto a planar surface.

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